Neural Networks and the Structure of the Proton

Andrea Piccione

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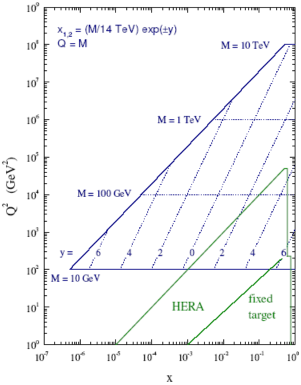
The NNPDF Collaboration

Luigi Del Debbio[1], Stefano Forte[2],
José I. Latorre[3], A. P.[4] and Joan Rojo[3]

[1] Theory Division, CERN
[2] Dipartimento di Fisica, Università di Milano and INFN, Sezione di Milano
[3] Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona
[4] Dipartimento di Fisica Teorica, Università di Torino and INFN, Sezione di Torino

What do we need for the LHC?



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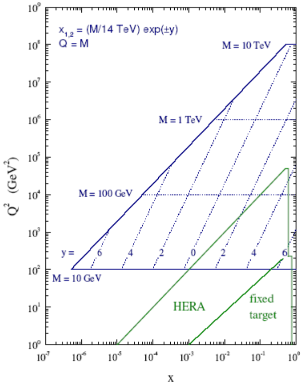

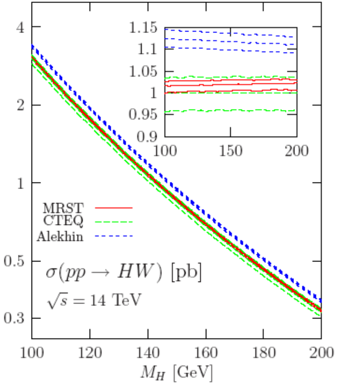

[Djouadi and Ferrag 2003]

Deep Inelastic Scattering

- The cross section

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[[1+(1-y)^2]F_1 + \frac{1-y}{x}\left(F_2 - 2xF_1\right)\right]$$

- The structure function

$$F_2(x,\ Q^2) = x\left[\sum_{q=1}^{n_f} e_q^2\, C^q \otimes q_q(x,\ Q^2) + 2n_f\, C^g \otimes g(x,\ Q^2)\right]$$

- Parton distribution evlution is described by DGLAP equations

$$Q^2\frac{d}{dQ^2}q(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}(P\otimes q)(x,Q^2)$$

Deep Inelastic Scattering and QCD

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The problem

- For a single quantity → 1 sigma error
- For a pair of numbers → 1 sigma ellipse
- For a function → We need an "error band" in the space of functions (*i.e.* the probability density $\mathcal{P}[f]$ in the space of functions $f(x)$)

Expectation values → Functional integrals

$$\langle \mathcal{F}[f(x)] \rangle = \int \mathcal{D}f \, \mathcal{F}[f(x)] \, \mathcal{P}[f(x)]$$

Determine an infinite-dimensional object (a function) from finite set of data points → Mathematically ill-posed problem

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The standard approach

1. Choose a simple functional form with enough free parameters

$$q(x, Q_0^2) = x^{\alpha}(1-x)^{\beta} P(x; \lambda_1, \ldots, \lambda_n)$$

2. Fit parameters by minimizing χ^2

Problem:

- Which is the theoretical bias due to the parametrization?

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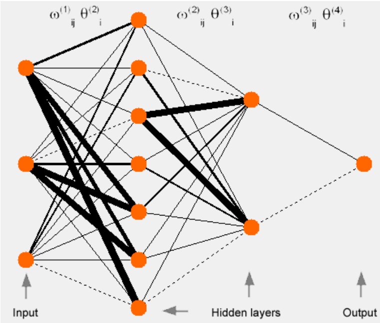
The NNPDF approach

- Determination of the Structure Functions:
  this is the easiest case, since no evolution is required, but only data fitting. A good application to test the technique $\longrightarrow$ Done
- Determination of the Parton Distributions:
  the real stuff $\longrightarrow$ Working on it ...

What are Neural Networks?

Neural networks are algorithms providing robust, universal, unbiased approximants to incomplete or noisy data



- Activation function:

$$\xi_i^{(l)} = g\left(\sum_{j=1}^{n_l-1} \omega_{ij}^{(l-1)}\xi_j^{(l-1)} - \theta_i^{(l)}\right)$$
$$g(x) = \frac{1}{1+e^{-x}}$$

- Parameters: weights $\omega_{ij}^{(l)}$ and thresholds $\theta_i^{(l)}$
- Assumption: smoothness and convergence criterium

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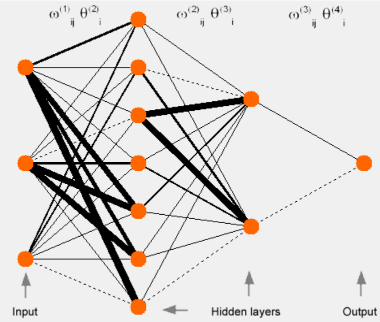

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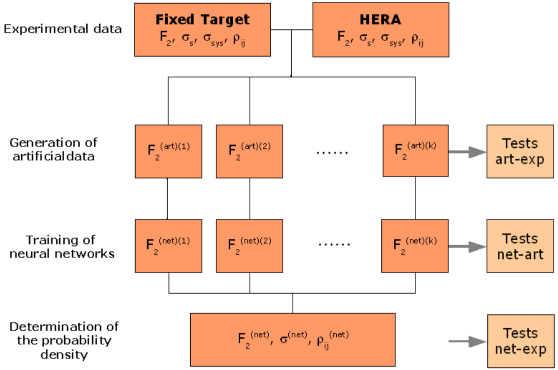
General strategy



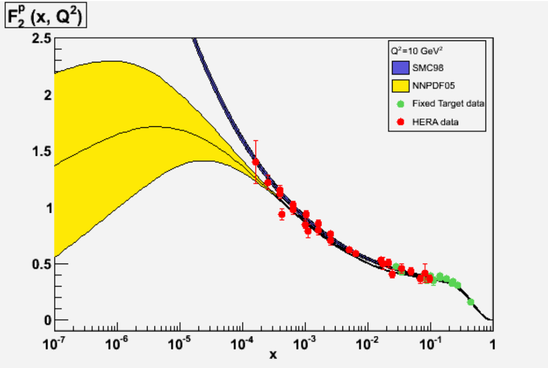
Credits

- S. Forte, L. Garrido, J. I. Latorre and A. P., *"Neural network parametrization of deep-inelastic structure functions,"* JHEP05 (2002) 062 [arXiv:hep-ph/0204232]
- L. Del Debbio, S. Forte, J. I. Latorre, A. P. and J. Rojo [NNPDF Collaboration], *"Unbiased determination of the proton structure function F_2^p with faithful uncertainty estimation"*, JHEP03 (2005) 080 [arXiv:hep-ph/0501067]

Source code, driver program and graphical web interface for F_2 plots and numerical computations available @

http://sophia.ecm.ub.es/f2neural

Fit of $F_2^p(x, Q^2)$ *[NNPDF 2005]*



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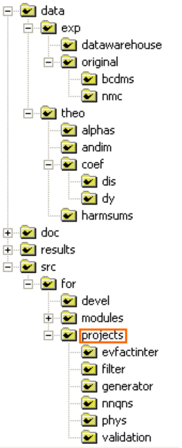
CVS tree:



Languages: F77, C++, ShellScript
Libraries: CERN, LHAPDF
#code lines: 9K
#files: 320
#folders: 115
#bytes: 14M

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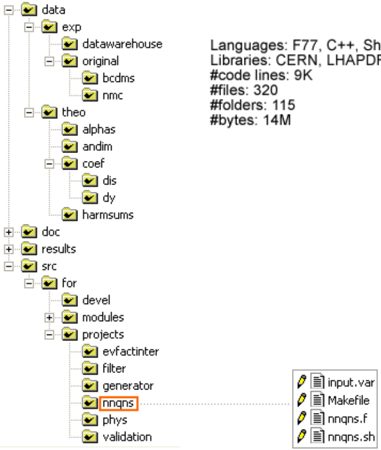

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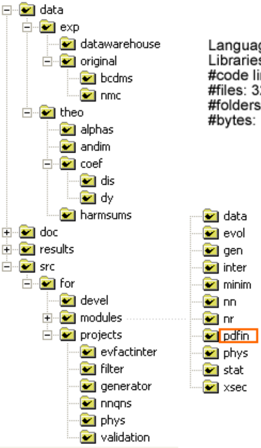

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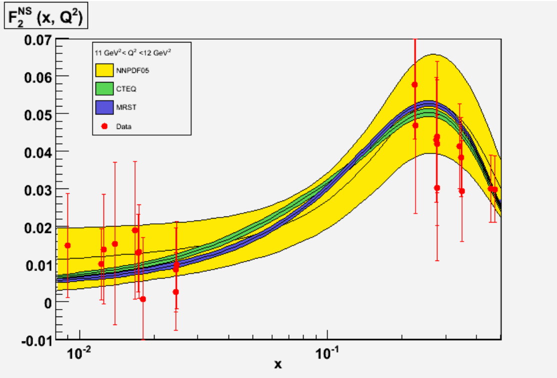
Non-Singlet


$F_2^{NS}(x, Q^2)$
11 GeV² < Q² < 12 GeV²
NNPDF05
CTEQ
MRST
Data
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
10^{-2}
10^{-1}
x

Outlook

- NN fit of other structure functions (*e. g.* $F_3(x, Q^2)$)
- Construct full set of NNPDF parton distributions from all available data
- Assess impact of uncertainties of PDFs for relevant observables at LHC
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