Latest results on PDF uncertainties with Neural Networks

NNPDF Collaboration

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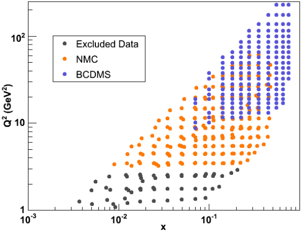
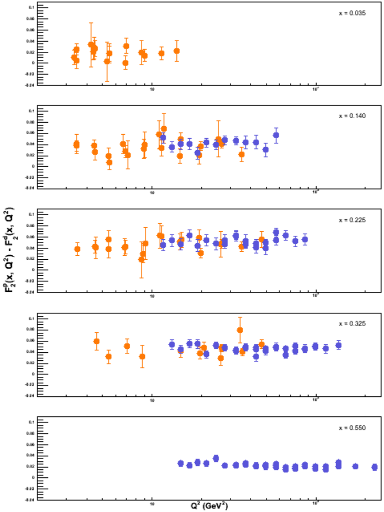
Some details





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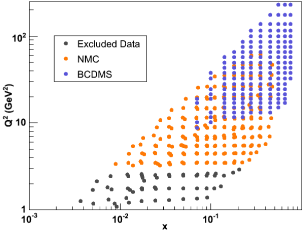

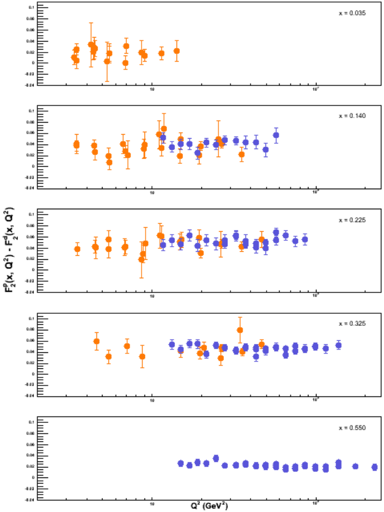

- $x\, q(x, Q_0^2) = NN(x, \log x)(1 - x)^a$
  $q(x, Q_0^2) = (u + \bar{u} - d - \bar{d})(x, Q_0^2)$
  $Q_0^2 = 2\ GeV^2$
- Neural network architecture: 2-5-3-1 (37 parameters).
- TMC: Georgi-Politzer,
  F_2 integral evaluated with NN F_2
- ZM-VFN:
  $m_c = 1.4 GeV$
  $m_b = 4.5 GeV$
  $m_t = 175 GeV$

First delivery

- 1000 MC reps
- LO ($\alpha_s = 0.130$), NLO ($\alpha_s = 0.118 \pm 0.002$), NNLO ($\alpha_s = 0.115$)
- LHAPDF interface
- With $\alpha_s = 0.118$ @ NLO we have:

	Total	NMC	BCDMS
χ^2/d.o.f.	0.95	0.92	0.97

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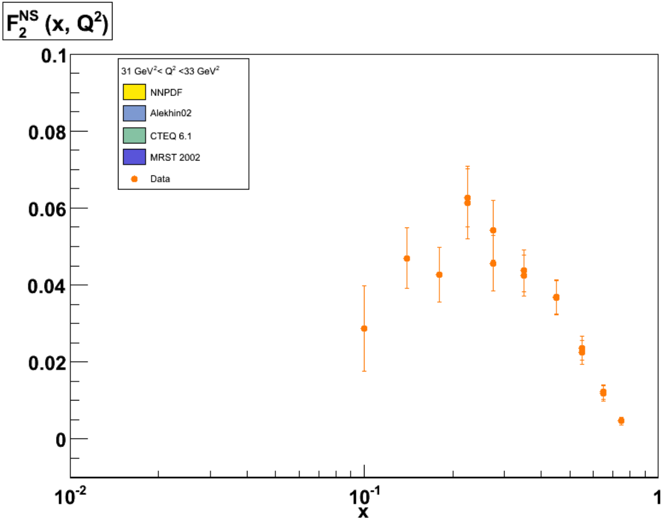
Reconstructing F_2^{NS} @ NLO with errors



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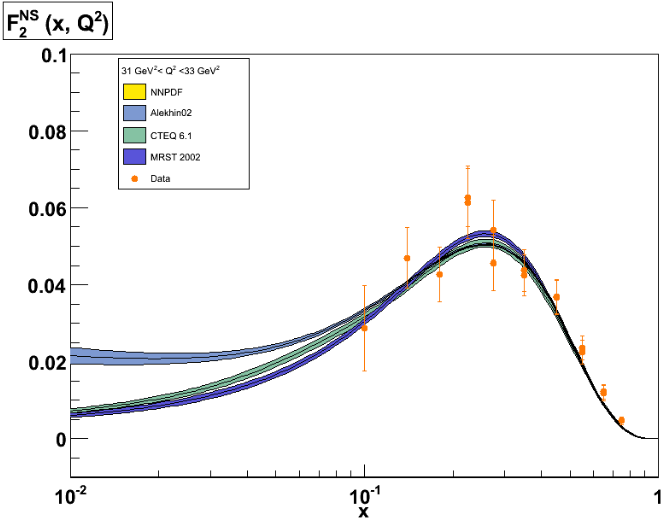

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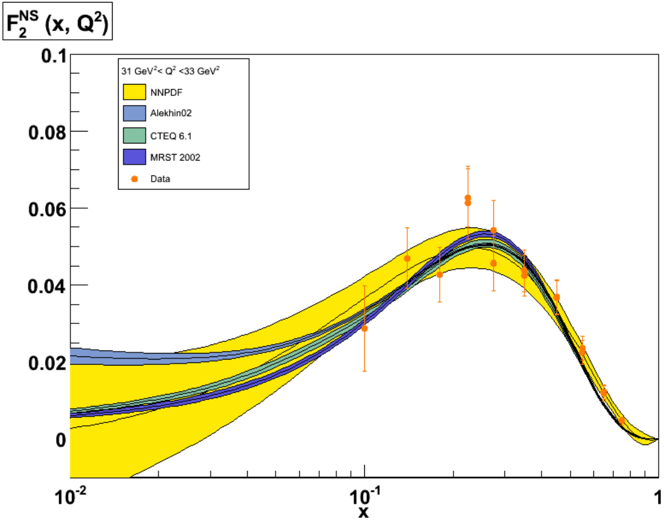

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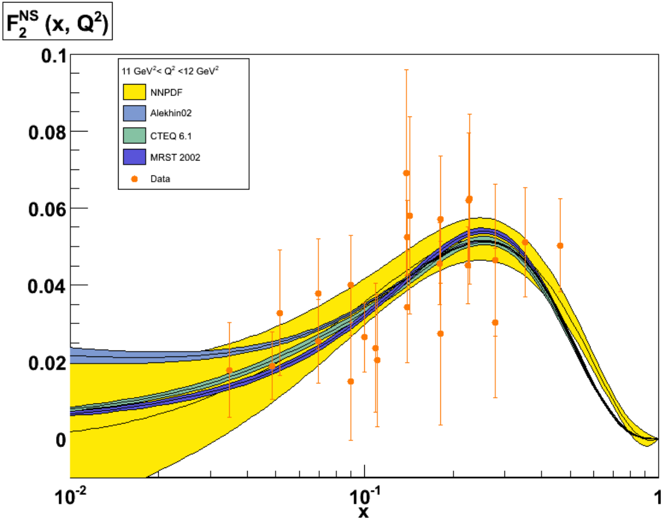
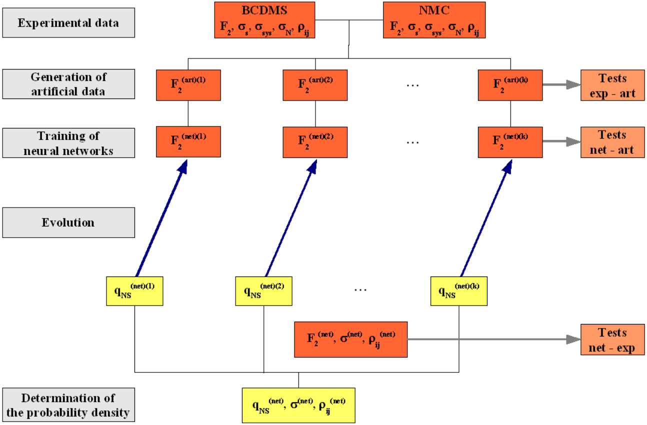


Schematically



Faithful error propagation: Data → Parametrization

- Monte Carlo sampling of data (generation of replicas of experimental data)

$$F_i^{(art)(k)} = \left(1 + r_N^{(k)}\sigma_N\right)\left[F_i^{(exp)} + r_i^s \sigma_i^{stat} + \sum_{l=1}^{N_{sys}} r^{l,(k)} \sigma_i^{sys,l}\right]$$

where σ_i are the experimantal errors, and r_i are random numbers choosen accordingly to the experimental correlation matrix.

Faithful error propagation: Parametrization → Observables

- Expectation values:

$$\langle \mathcal{F}[g(x)] \rangle = \frac{1}{N_{rep}} \sum_{k=1}^{N_{rep}} \mathcal{F}\left(g^{(net)(k)}(x) \right)$$

- Errors:

$$\sigma_{\mathcal{F}[g(x)]} = \sqrt{\left\langle \mathcal{F}[g(x)]^2 \right\rangle - \langle \mathcal{F}[g(x)] \rangle^2}$$

- Correlations between pairs of different parton distributions at different points:

$$\langle u(x_1) d(x_2) \rangle = \frac{1}{N_{rep}} \sum_{k=1}^{N_{rep}} u^{(net)(k)}(x_1, Q_0^2) d^{(net)(k)}(x_2, Q_0^2)$$

Unbiased parametrization

- Activation function ("a smooth step"):

$$\xi_i^{(l)} = g\left(\sum_{j=1}^{n_l-1} \omega_{ij}^{(l-1)} \xi_j^{(l-1)} - \theta_i^{(l)}\right), \qquad g(x) = \frac{1}{1+e^{-x}}$$

- As an example, in a very simple case (1-2-1) we have

$$\xi_1^{(3)} = \frac{1}{1 + e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1+e^{\theta_1^{(2)} - \xi_1^{(1)}\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1+e^{\theta_2^{(2)} - \xi_1^{(1)}\omega_{21}^{(1)}}}}}$$

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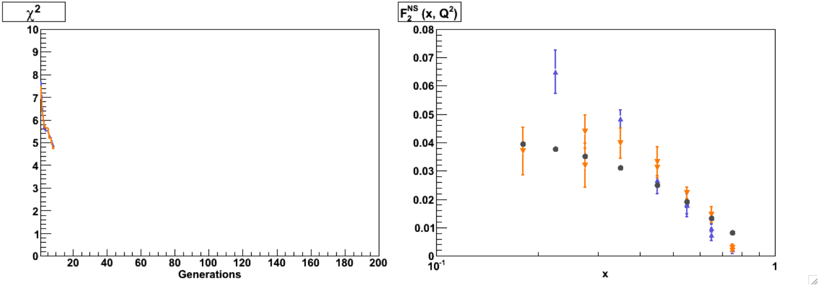
Minimization with a Genetic Algorithm

1. Set the parameters randomly.
2. Make clones of the set of parameters.
3. Mutate randomly each clone.
4. Evaluate χ^2 for all the clones.
5. Select clones with the lowest χ^2.
6. Back to 2, till $\chi^2 \sim \bar{\chi}^2$.

An example of learning a replica of data



- We divide data in a training and in a validation set.

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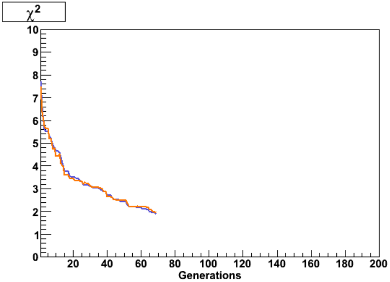

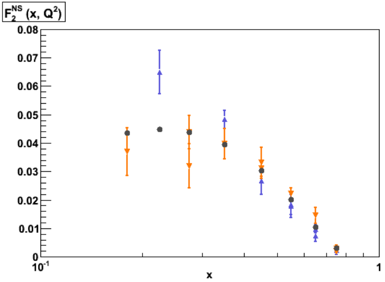

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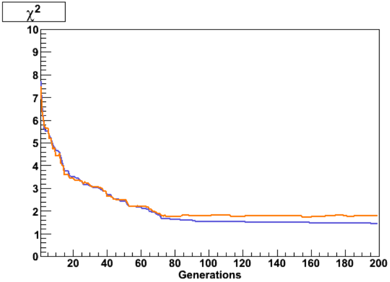

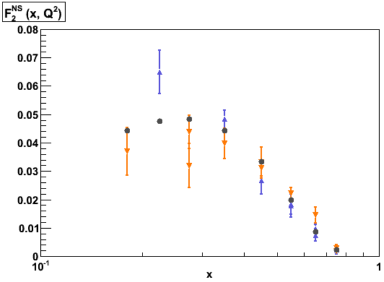

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We have developed a tool to fit data that

- provides a faithful combination of experimental errors;
- allows a faithful propagation of errors on computed observables;
- handles incompatibilities among experiments without assumptions;
- avoids theoretical biases on the used parametrization.

Perspectives:

- a singlet set from DIS data (December 2006?)
- a singlet set from DIS+DY data (April 2007?)

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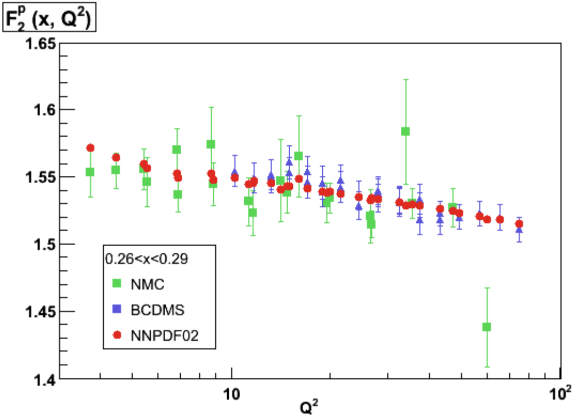
Incompatible data

[S. Forte et al., hep-ph/0204232 - A. P., hep-ph/0207204]



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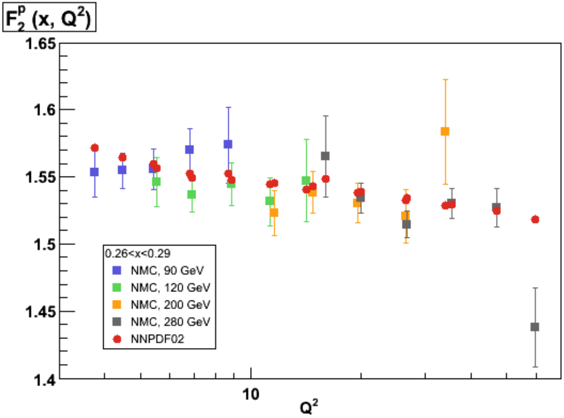

A new framework

- We want Mellin space evolution:

$$q(N, Q^2) = q(N, Q_0^2)\Gamma\left(N, \alpha_s\left(Q^2\right), \alpha_s\left(Q_0^2\right)\right)$$

- We do not want complex neural networks:

$$\Gamma\left(x, \alpha_s\left(Q^2\right), \alpha_s\left(Q_0^2\right)\right) \equiv \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, x^{-N}\Gamma\left(N, \alpha_s\left(Q^2\right), \alpha_s\left(Q_0^2\right)\right)$$

- The evolved PDF is given by

$$q(x, Q^2) = \int_x^1 \frac{dy}{y}\Gamma\left(y, \alpha_s\left(Q^2\right), \alpha_s\left(Q_0^2\right)\right) q\left(\frac{x}{y}, Q_0^2\right)$$